Math Packets Summer

This packet is intended for students going into

Algebra 1

Directions: Complete the following math packet week by week. Each week you will find the topic divided into parts so you can manage the workload. This packet has 6 weeks of materials. Take your time and avoid the summer slide by completing the following work that will prepare you for Algebra 1. Additionally, at the end of each section, you will find a “Minute math” activity. These problems are designated to improve your math fluency and practice using strategies for solving a variety of problems.

Week 1:

Part 1:

• Addition and Subtraction Word Problems

To solve word problems:

1. Identify the pattern or plot.
2. Write an equation for the given information.
3. Solve for the unknown number.
4. Check to see if the answer makes sense.

Combining (some, some more) $s + m = t$	some ⟵ Find by subtracting + more ⟵ Find by subtracting total ⟵ Find by adding
Separating (some went away) $s - a = l$	starting amount ⟵ Find by adding − some went away ⟵ Find by subtracting what's left ⟵ Find by subtracting
Comparing (greater/less) $g - l = d$	greater ⟵ Find by adding − lesser ⟵ Find by subtracting difference ⟵ Find by subtracting
Elapsed Time $l - e = d$	later ⟵ Find by adding − earlier ⟵ Find by subtracting difference ⟵ Find by subtracting

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Practice:

1. Amy went to the store with $10.00 and came home with $4.65. How much money did Amy spend at the store? ____________________

2. Danny collected 38 cans at the playground. He then collected 27 cans at the ball field. How many cans did Danny collect in all? ____________________

3. From 2000 to 2005, the population of Red Valley School increased from 487 students to 732 students. How many more students were in Red Valley School in 2005 than in 2000? ____________________

4. The soccer team sold 187 flags on Saturday. For the whole weekend, they sold 265 flags. How many flags did the soccer team sell on Sunday? __________

5. Which equation shows how to find what year someone was born when you know that they were 81 in 2005? Circle the correct answer.

a. $81 - b = 2005$ **b.** $2005 + 81 = b$ **c.** $b - 2005 = 81$ **d.** $2005 - b = 81$

Part 2:

• Multiplication and Division Word Problems

To solve word problems:

1. Identify the pattern or plot.
2. Write an equation for the given information.
3. Solve for the unknown number.
4. Check to see if the answer makes sense.

Equal Groups		
$n \times g = t$	number of groups ←	Find by dividing
	× number in group ←	Find by dividing
	total ←	Find by multiplying

Example: Cory sorted 375 quarters into groups of 40 so that he could put them in rolls. How many rolls can Cory fill with quarters? Explain why your answer is reasonable.

Write the equation: $n \times 40 = 375$
Find the unknown: $375 \div 40 = 9$ R 15
Cory can fill 9 rolls with 15 quarters remaining.
Our answer is reasonable because 9 rolls of 40 quarters would be 360, whereas 10 rolls of 40 quarters would equal 400 quarters. So 375 quarters is not enough to fill 10 rolls but is enough to fill 9 rolls, leaving 15 quarters unrolled.

Practice:

1. Emma arranged the cards in 9 rows with 18 cards in each row. How many cards did Emma arrange? ____________________

2. Jordan had 876 pennies. How many rolls of 50 pennies can he fill? Explain why your answer is reasonable. ____

____

3. The muffins were arranged in rows with 12 muffins in each row. If there were 96 muffins, how many rows of muffins were there? ____________________

4. Three student tickets to the water park cost $77.25. The cost of each ticket can be found using which of the following equations? Circle your answer.

a. $\$77.25 \times 3 = t$ **b.** $3 \times t = \$77.25$

c. $\$77.25 \times t = 3$ **d.** $\frac{t}{3} = \$77.25$

Minute 1

1. $2^3 =$

2. $27 \div 9 + 3 =$

3. If $m + 40 = 75$, then $m =$ __________.

4. Number of letters in the alphabet minus the number of months in a year? __________

5. $(4 + 2)^2 =$

6. Write $3 \cdot 3 \cdot 3 \cdot 3$ in exponential form. __________

7. $8 \cdot 9 =$

8. $\frac{48}{6} =$

9. $1^{10} =$

10. $5 + (4)(3) =$

Bonus! Farmer Doug has some pigs and chickens. One day he counted 24 legs and 7 heads in the barnyard. How many of each animal did Farmer Doug count? __________

Week 2:

Part 1:

- **Evaluation**
- **Solving Equations by Inspection**

- A **formula** is an expression or equation. To use a formula, substitute the number you know to find the number you want to know.

  Example: Use the formula $P = 4s$ to find the perimeter of a square with sides 6 cm long.

  $P = 4s \longrightarrow$ substitute 6 cm for $s \longrightarrow P = 4(6) = 24$ cm

- To solve an equation, find the number for the variable that makes the equation true.

  Example: $4x = 20 \longrightarrow$ The solution is 5 because $4(5) = 20$.

- To **solve equations by inspection**, study equations to mentally find the solutions.

Practice:

1. A formula for area (A) of a parallelogram is $A = bh$. Find the area of a parallelogram with a base (b) of 15 cm and a height (h) of 9 cm.

   ____

2. Evaluate $2ac$ for $a = 4$ and $c = 24$. ______________________________

Solve each equation in problems 3–5 by inspection.

3. $6 + x = 32$

   $x =$ ______________________________

4. $k - 8 = 14$

   $k =$ ______________________________

5. $\frac{m}{9} = 7$

   $m =$ ______________________________

6. Admission to the carnival is \$2. Each game ticket costs \$2. Brianna has \$10. Solve the following equation to find how many game tickets Brianna can buy.

   $$2t + 2 = 10$$

   ____

Part 2:

• Powers and Roots

- An **exponent** shows repeated multiplication.

$$\text{base} \rightarrow 5^3 \leftarrow \text{exponent}$$

It shows how many times the base is used as a factor. Write 5 **three** times then **multiply**.

$$5^3 = 5 \cdot 5 \cdot 5 = 125$$

- Like factors can be grouped with an exponent.

Example: The prime factorization of 72 is $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$.

$$72 = 2^3 \cdot 3^2$$

- The exponent 2 used after a unit of length is a unit of area.

9 in.^{2} means 9 square inches

- Exponents can be applied to variables.

Example: $2xxyyyz = 2x^2y^3z$

- The inverse of raising a number to a power is taking a root of a number.

$\sqrt{25} = 5$ "The square root of 25 is 5."
$\sqrt[3]{125} = 5$ "The cube root of 125 is 5."

Practice:

1. Simplify: $7^3 =$ ____________________

2. Simplify: $5^4 =$ ____________________

3. Simplify: $\sqrt{16} =$ ____________________

4. Simplify: $\sqrt[3]{27} =$ ____________________

5. Express with exponents: $4xyyxx =$ ____________________

6. Mr. Ortiz wants new flooring for his office. His office is 12 feet square. Use the formula $A = s^2$ to find the number of square feet of flooring he needs.

____

MINUTE 2

1. (2)(3)(4) =

2. Write 4 • 4 • 4 • 4 • 4 in exponential form. __________

3. $\frac{4 + 6}{5} =$

4. Bobby thinks that $5^2 = 10$.
What is wrong with this answer? ______________________________

5. 4 + 6 • 2 = 4 + 12 Circle: True or False

6. If $a = 5$ and $b = 6$, then what does ab equal? __________

7. Miss White wants to buy 5 value meals at Mel's Diner.
What is a reasonable total for her purchase?
a. \$25 **b.** \$1,000 **c.** \$100 **d.** \$10

8. 12 snakes have how many eyes altogether? __________

9. 5 + (9)(6) =

10. Which of these operations should be completed first when solving an equation?
a. × **b.** + **c.** ( ) **d.** ÷

Week 3:

Part 1:

- **Distributive Property**
- **Order of Operations**

Distributive Property
$a(b + c) = a \cdot b + a \cdot c$
$a(b + c + d) = a \cdot b + a \cdot c + a \cdot d$
$a(b - c) = a \cdot b - a \cdot c$

- We **expand** $2(a + b)$ and get $2a + 2b$.
- We **factor** $2a + 2b$ and get $2(a + b)$.

Examples: Expand: $3(x + 2) = 3x + 6$

Factor: $6x + 9$ ← divide to remove a common factor from each term

$3(2x + 3)$

- If there is more than one operation, follow this order:
  1. Simplify within **parentheses**.
  2. Simplify **exponent** expressions.
  3. **Multiply** and **divide** in order from left to right.
  4. **Add** and **subtract** in order from left to right.
- Remember this order as: "**P**lease/**e**xcuse/**m**y **d**ear/**A**unt **S**ally."
- Follow the order of operations to simplify within the parentheses.
- Simplify expressions with multiple grouping symbols beginning from the innermost symbol.

Practice:

Expand.

1. $4(2 + y) =$ ____________________

2. $5(x - 3) =$ ____________________

Factor.

3. $16w + 12 =$ ____________________

4. $6x - 15 =$ ____________________

Simplify.

5. $3 + 4 \cdot 5 - 6 =$ ____________________

6. $(36 \div 6) \cdot (12 - 6) =$ ____________________

7. $2^3 \cdot (14 - 9) + 8 =$ ____________________

8. $20 - [3 \cdot (14 \div 7)] =$ ____________________

Part 2:

• Multiplying and Dividing Fractions

- To multiply fractions, multiply numerators, and then multiply denominators.

  Example: $\frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9}$

- To find $\frac{3}{4}$ of $\frac{8}{9}$, multiply the fractions. We may reduce (cancel) before we multiply.

$$\frac{{}^{1}3}{{}_{1}4} + \frac{8^{2}}{9_{3}} = \frac{2}{3}$$

- If the product of two fractions is 1, the fractions are **reciprocals**.

  Example: $\frac{2}{3} \cdot \frac{3}{2} = \frac{6}{6} = 1 \longleftarrow$ $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals

- Form the reciprocal of a fraction by reversing the numbers in the numerator and denominator, or inverting the fraction.
- Use reciprocals to help divide fractions.

Example: $2 \div \frac{3}{4}$

$\downarrow \quad \downarrow \quad \downarrow$

$\frac{2}{1} \times \frac{4}{3} = \frac{8}{3} = 2\frac{2}{3}$

Example: How many $\frac{1}{8}$s are in $\frac{1}{4}$?

$\frac{1}{4} \div \frac{1}{8}$

$\frac{1}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{1} = \frac{2}{1} = 2$

Practice:

1. $\frac{1}{3} \times \frac{2}{5} =$ ____________________

2. $\frac{3}{8} \cdot \frac{4}{5} =$ ____________________

3. $1 \div \frac{2}{3} =$ ____________________

4. $\frac{1}{2} \div \frac{3}{4} =$ ____________________

5. What number is $\frac{1}{2}$ of $\frac{3}{8}$? ____________________

6. How many $\frac{5}{8}$ are in $\frac{3}{4}$? ____________________

MINUTE 3

1. $2(5 + 8) =$

2. Rewrite 4 • 4 • 6 • 4 • 4 • 6 using exponents. __________

3. $\frac{3(4+2)}{9} =$

4. Brad thinks that 2 • 2 • 2 • 2 is represented by 4^2.
What is wrong with this answer? ______________________________

5. $3.2 \times 10^3 =$

6. If $a = 2$ and $b = 3$, then what does ab^2 equal? __________

7. $0.043 \times 10^3 =$

8. A mouse has 14 whiskers.
How many whiskers do 3 mice have? __________

9. $5 + (9)(6) - 4 =$

10. Which of these operations should be completed <u>last</u> when solving an equation?

a. × **b.** + **c.** ( ) **d.** ÷

Week 4:

Part 1:

• Laws of Exponents

- A counting-number exponent indicates how many times a base is a factor.

  Example: $x^4 = x \cdot x \cdot x \cdot x$

- Laws of Exponents describe relationships between exponents for certain operations.

Laws of Exponents for Multiplication and Division

Law	Description	Example
$x^a \cdot x^b = x^{a+b}$	When the bases are the same, and you multiply, you add the exponents.	$x^5 \cdot x^3 = x^8$
$\frac{x^a}{x^b} = x^{a-b}$	When you divide, you subtract the exponents.	$\frac{x^5}{x^3} = x^2$
$(x^a)^b = x^{ab}$	When you raise a term with an exponent to a power, you multiply the exponents.	$(x^3)^2 = x^6$

Practice:

1. $x^6 \cdot x^2 =$ ____________________

2. $\frac{x^4}{x^2} =$ ____________________

3. $(x^4)^3 =$ ____________________

4. $2^4 \cdot 2^2 =$ ____________________

5. $(2^2)^3 =$ ____________________

6. Write this expression as a power of 10.

   $\frac{10^6}{10^3} =$ ____________________

Part 2:

• Ratio

- A **ratio** is a comparison of two numbers by division.

  For example, if there are 16 girls and 12 boys in the class, the ratio of girls to boys can be expressed as a reduced fraction.

$$\frac{\text{Girls}}{\text{Boys}} = \frac{16}{12} = \frac{4}{3}$$

- Write the ratio in the order stated
- Write the ratio as a reduced fraction, but not as a mixed number.
- The ratio of "3 to 4"can be written four ways:

with the word "to"	3 to 4
as a fraction	$\frac{3}{4}$
as a decimal number	0.75
with a colon	3:4

- We can round large numbers and reduce the ratio.

  Example: $\frac{\text{1217 home fans}}{\text{897 visiting fans}} \approx \frac{1200}{900} = \frac{4}{3}$

- A rate is a ratio of two measures with different units. The units do not cancel, but remain part of the rate.

  Example: $\frac{\text{35 miles}}{\text{gallon}} \longrightarrow$ 35 miles per gallon

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Practice:

Use for Problems 1 and 2: There are 32 students in the math class. Fourteen students are boys and eighteen students are girls.

1. What is the ratio of boys to girls? __________

2. What is the ratio of girls to boys? __________

Use for Problems 3 and 4: A 25-foot tree casts a shadow 35 feet long.

3. What is the ratio of the height of the tree to the length of the shadow? __________

4. What is the ratio of the length of the shadow to the height of the tree? __________

5. Mrs. Shaw corrected 25 tests in 5 minutes. Find the rate of test corrections per minute. ______________________________

6. If 1487 adults and 912 children attended the concert, what was the approximate ratio of adults to children? ______________________________

MINUTE 4

1. $3.57 \times 10^3 =$

2. $2^2 \bullet 2^3 =$

3. Which of these represents a whole number?
Circle all that apply.
a. 4 **b.** 3.2 **c.** $\frac{4}{7}$ **d.** $\frac{8}{4}$

4. Which of these represents an integer?
Circle all that apply.
a. –3 **b.** 4 **c.** $2\frac{1}{2}$ **d.** 6.2

5. Which expression is correctly written in scientific notation?
a. 398×10^1 **b.** 39.8×10^2
c. 3.98×10^4 **d.** $.398 \times 10^3$

6. $\frac{8 + 4 \bullet 3}{5} =$

7. $2^{-2} =$

8. $\frac{3^3}{3^2} =$

9. $\sqrt{25} =$

10. $3(4^2 + 1) =$

Week 5: Part 1:

• Adding Integers

Integers include positive numbers, negative numbers, and zero. When we add two integers, the sign of the sum depends on the sign of both addends.

- The sum of **two positive** integers is always **positive.**
  Example: (+4) + (+4) = +8

- The sum of **two negative** integers is always **negative.**
  Example: (–5) + (–3) = –8

- The sum of two integers with different signs may be positive, negative, or zero.

- The sum of any integer and its opposite is always zero.
  Examples: (–2) + (+2) = 0 (+5) + (–5) = 0

- The sign of the sum of a positive integer and a negative integer that are not opposites depends on their absolute values. Remember: **Absolute value** is the distance from zero on the number line.

- To find the sum of two integers with different signs:
  1. Subtract the absolute values of the addends.
  2. Take the sign of the addend with the greater absolute value.

  Example 1: Find the sum. (–2) + (+7)
  First subtract the absolute values. |–2| = 2 |+7| = 7
  7 – 2 = 5
  Since the absolute value of +7 is greater than the absolute value of –2, the sign of the sum is positive. (–2) + (+7) = +5

- The sign of the sum of a positive integer and a negative integer that are not opposites depends on their absolute values. Remember: **Absolute value** is the distance from zero on the number line.

- To find the sum of two integers with different signs:
  1. Subtract the absolute values of the addends.
  2. Take the sign of the addend with the greater absolute value.

  Example 1: Find the sum. (–2) + (+7)
  First subtract the absolute values. |–2| = 2 |+7| = 7
  7 – 2 = 5
  Since the absolute value of +7 is greater than the absolute value of –2, the sign of the sum is positive. (–2) + (+7) = +5

  Example 2: Find the sum. (+2) + (–7)
  First subtract the absolute values. |+2| = 2 |–7| = 7
  7 – 2 = 5
  Since the absolute value of –7 is greater than the absolute value of +2, the sign of the sum is negative. (+2) + (–7) = –5

Practice:

Simplify.

1. (–12) + (+4) = ______

2. (+27) + (+3) = ______

3. (–6) + (–7) = ______

4. (–8) + (+8) = ______

5. (–3) + (+9) = ______

6. (+15) + (–19) = ______

Part 2:

• Subtracting Integers

Subtracting an integer is the same as adding its opposite.
The opposite of a positive integer is a negative integer with the same absolute value.
The opposite of a negative integer is a positive integer with the same absolute value.
The opposite of any integer is called the **additive inverse.**

Examples: The additive inverse of 4 is -4. The additive inverse of -6 is 6.

To subtract integers, replace the subtrahend with its opposite and then add.

Examples:

$$-3 - (-8) \;\rightarrow\; -3 + (+8) = \mathbf{5}$$

$$-7 - (+3) \;\rightarrow\; -7 + (-3) = \mathbf{-10}$$

Addition and Subtraction with Two Integers

To find the sum of addends with different signs: 1. Subtract the absolute values of the addends. 2. Take the sign of the addend with the greater absolute value.
To find the sum of addends with the same sign: 1. Add the absolute values of the addends. 2. Take the sign of the addends.
Instead of subtracting a number, add its opposite.

Practice:

Simplify.

1. $(-15) - (+4) =$ ____________

2. $(+7) - (+13) =$ ____________

3. $(-9) - (-5) =$ ____________

4. $(-8) - (+8) =$ ____________

5. $(-3) - (-4) =$ ____________

6. $(+10) - (-17) =$ ____________

MINUTE 5

1. $3^2 \cdot 3 \cdot 3 \cdot 3 = 3^4$ Circle: True or False

2. Write 5,806 in scientific notation. __________

3. $2^{-3} = \frac{1}{2^3}$ Circle: True or False

4. $\sqrt{64} =$

5. $3[8 + (4 + 2)] =$

6. What does a equal in this problem? $8 = 2^a$ __________

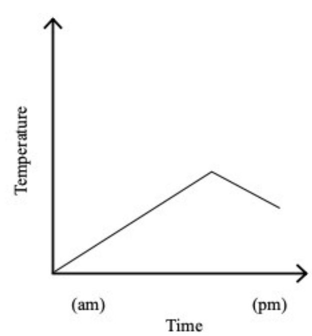
7. $x \cdot x \cdot x =$

8. If $a = 6$ and $b = 2$, then what does a^b equal? __________

9. $\frac{2^5}{2^3} =$

10. According to the graph, which of these is true?

a. The later in the day it is, the hotter it is.
b. Temperature goes up and then down during the day.
c. Temperature is always lowest in the evening.
d. Temperature decreases when it rains.



Week 6: Part

1:

- **Proportions**
- **Ratio Word Problems**

- A **proportion** tells us that two ratios are equal.

  Examples: $\frac{4}{20} = \frac{20}{100}$ $\frac{1}{2} = \frac{2}{4}$

- You can check to see if two ratios are equal by reducing each ratio to simplest form. Equal ratios reduce to the same fraction.

  Example: The proportion $\frac{6}{16} = \frac{12}{32}$ is true because $\frac{6}{16} = \frac{3}{8}$ and $\frac{12}{32} = \frac{3}{8}$.

- Another way to verify that a proportion is true is to find a constant factor for the numerator and the denominator that changes one ratio to the other.

  Example: $\frac{6}{16} = \frac{12}{32}$ because $\frac{6 \cdot 2}{16 \cdot 2} = \frac{12}{32}$.

- If you can identify a constant factor, you can find a missing number in a proportion.

  Examples:

  $\frac{5}{60} = \frac{x}{180}$

  $\frac{5 \cdot 3}{60 \cdot 3} = \frac{15}{180}$

  $\frac{35}{10} = \frac{7}{x}$

  $\frac{35 \div 5}{10 \div 5} = \frac{7}{2}$

- You can use a ratio box to help you solve ratio word problems.

  Example: The ratio of cats to dogs is 4 to 9. If there are 12 cats, how many dogs are there?

	Ratio	Actual
Cats	4	12
Dogs	9	x

$\frac{4}{9} = \frac{12}{x}$ $\left(\frac{4}{9}\right) \cdot \left(\frac{3}{3}\right) = \frac{12}{27}$

$x = 27$, so there are 27 dogs.

Practice:

Solve each proportion.

1. $\frac{3}{9} = \frac{15}{x}$ ____________

2. $\frac{x}{10} = \frac{16}{40}$ ____________

3. $\frac{6}{12} = \frac{3}{x}$ ____________

4. Two pounds of apples cost \$3. How much do 6 pounds cost? ________

Part 2:

• Using Properties of Equality to Solve Equations

When you solve an equation, you must always perform the same operation on both sides of the equation to keep it balanced.

Inverse operations can help you solve an equation by **isolating the variable** on one side of the equation.

Addition and subtraction are inverse operations. $n + 3 - 3 = n$

Multiplication and division are inverse operations. $n \div 3 \cdot 3 = n$

Example:

Solve the equation. $y - 1.8 = 3.4$

Recall that inverse operations "undo" each other.

So to undo subtracting 1.8 from y, we add 1.8 to y. $y - 1.8 + 1.8 = 3.4 + 1.8$

To keep the equation balanced, we also add 1.8 to 3.4.

Simplify both sides to solve for y. $y = 5.2$

Example:

Solve the equation. $3 = \frac{3}{4}y$

To undo multiplying y by $\frac{3}{4}$, we divide y by $\frac{3}{4}$.

Recall that dividing by $\frac{3}{4}$ is the same as multiplying by $\frac{4}{3}$. To keep the equation balanced, we also multiply 3 by $\frac{4}{3}$. $3 \cdot \frac{4}{3} = \frac{3}{4}y \cdot \frac{4}{3}$

$4 = y$

Simplify both sides to solve for y.

Reverse the equation, using the Symmetric Property. $y = 4$

Practice:

Solve each equation.

1. $x + \frac{1}{3} = \frac{7}{9}$ ______

2. $8m = 6.4$ ______

3. $35 = \frac{r}{0.7}$ ______

4. $x - 12 = 10.5$ ______

5. $\frac{7}{8} = \frac{1}{2}x$ ______

6. $8t = 60$ ______

7. $w - \frac{1}{3} = \frac{1}{2}$ ______

8. $\frac{2}{3}x = 6$ ______

9. $1.2 + y = 4$ ______

MINUTE 6

1. $2\sqrt{49} =$

2. Write 20,136 in scientific notation. __________

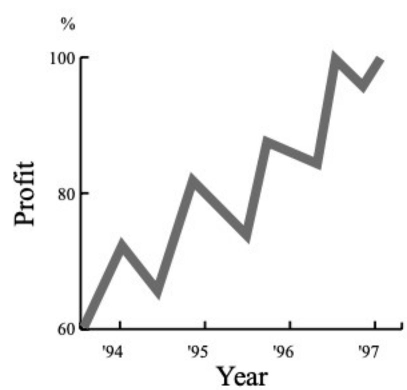
3. $\left(\frac{2}{3}\right)^2 =$

4. $2^2 \cdot 2^2 = 2^4$ Circle: True or False

5. $\frac{2 \cdot 3 \cdot 4}{2 \cdot 3} =$

6. $\sqrt{16} \cdot \sqrt{25} =$

7. According to the graph on the right,
would it be a good idea to invest in this company?
Circle: Yes or No



8. $[(2 + 3) \cdot 4] =$

9. $\frac{4^6}{4^4} =$

10. $3^{-2} =$

BONUS! The sum of two numbers is 9 and their difference is 3.
What is their product? __________

Have a good Summer!